# 1. SUMMARY

The principal subject of the research is the study of spaces which arise from well-behaved actions of the compact torus $T^n$.

Since the 1970s, the study of torus actions has become increasingly important in various areas of pure mathematics, and has stimulated the formation of interdisciplinary links between algebraic geometry, combinatorial and convex geometry, commutative and homological algebra, differential topology and homotopy theory. As their net has spread wider, and the literature grown, a field of activity has emerged which merits the title *toric topology*. Toric topology is the study of algebraic, combinatorial, differential, geometric, and homotopy theoretic aspects of a particular class of torus actions, whose quotients are highly structured. A characteristic feature is the calculation of invariants in terms of combinatorial data associated to the quotients; a primary goal is to classify toric spaces by means of these invariants.

The initial impetus for these developments was provided by the theory of *toric varieties* in algebraic geometry. Since the beginning of the 1990s the ideas and methodology of toric varieties have started penetrating back into algebraic topology. When a torus $T^n$ acts nicely on a topological space $X$, the orbit space admits a rich combinatorial structure reflecting the distribution of the isotropy subgroups. Not only does this structure provide a powerful means of investigating the action, but some of its subtler combinatorial properties may also be illuminated by the equivariant topology of $X$. Stanley was one of the first to realise the full potential of the subject for combinatorial applications, using it to prove McMullen's conjecture for face vectors of simplicial polytopes, and the Upper Bound Conjecture for triangulated spheres. Further developments reveal two particular classes of $T^n$-manifolds, both taking their origins from toric varieties, which provide an illuminating context for combinatorial and algebraic applications of toric topology. These are the *quasitoric manifolds* of Davis–Januszkiewicz, and the *torus manifolds* of Hattori–Masuda. The work of Buchstaber–Ray reveals that toric manifolds are influential players in the complex cobordism theory, a classical subject in algebraic topology. *Moment-angle complexes* $\mathcal{Z}_K$, studied in the work of Buchstaber–Panov, constitute another important family of toric spaces associated with simplicial complexes $K$. They arise in homotopy theory as homotopy colimits, in symplectic topology as level surfaces for the moment maps of Hamiltonian torus actions, and in the theory of arrangements as complements of coordinate subspace arrangements. According to a result of Buchstaber–Panov, the cohomology of the moment-angle complex is isomorphic to the Tor-cohomology of the *Stanley–Reisner ring* of the associated simplicial complex.

In the current project we plan to concentrate on the following three particular aspects of toric topology.

1. Continue the study of quasitoric manifolds in the context of complex cobordisms, initiated in the work of Buchstaber–Ray and Buchstaber–Panov–Ray. An ultimate aim here is to construct a purely combinatorial model for the complex cobordism theory.

2. Homotopy theoretical aspects of toric topology. Analyse the homotopy type of the moment-angle complex $\mathcal{Z}_K$ and its loop space by constructing appropriate algebraic models. Apply these models to calculate the Ext-cohomology (Yoneda algebras) of the Stanley–Reisner rings. Investigate combinatorial applications to problems related to the face numbers of simplicial complexes.

3. Bott towers, or iterated $\mathbb{C}P^1$-bundles over $\mathbb{C}P^1$ constitute a particularly important family of toric manifolds. By extending the results of Masuda–Panov, we plan to obtain a topological classification of Bott towers in terms of their cohomology rings.

Toric spaces and associated objects, besides being of considerable topological interest, bind together such fields as combinatorial geometry, commutative and homological algebra, and the theory of configuration spaces and arrangements. Any significant breakthrough in our understanding of the topology of these spaces will lead instantly to a host of applications in all of these areas.

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To be submitted together with T. Panov's application for P. Deligne contest.